# III Semester B.A./B.Sc. Examination, Nov./Dec. 2014
## (Semester Scheme) (N.S.)
## (2012-13 and Onwards)
## MATHEMATICS – III

Time : 3 Hours Max. Marks : 100

***Instruction :*** *Answer* ***all*** *questions.*

I. Answer **any fifteen** questions : **(15×2=30)**

1) If H is a normal subgroup of a group G, then prove that the product of any two right cosets of H in G is again a right coset of H in G.

2) Define kernel of a homomorphism.

3) If $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 4 & 2 \end{pmatrix}$ $\beta = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$ find $\alpha \circ \beta^{-1}$.



4) Let $f : G \to G'$ be a homomorphism from the group G into a group $G'$ with kernel K then prove that f is one-one if $K = \{e\}$.

5) Using column minima method determine an initial basic solution of the following transportation problem.

| Origin \ Destination | $D_1$ | $D_2$ | $D_3$ | $D_4$ | Availability |
|---|---|---|---|---|---|
| $O_1$ | 1 | 2 | 1 | 4 | 30 |
| $O_2$ | 3 | 3 | 2 | 1 | 50 |
| $O_3$ | 4 | 2 | 5 | 9 | 20 |
| Requirement | 20 | 40 | 30 | 10 | 100 |

6) Solve $3x + 5y \leq 15$ graphically.

7) Define basic solution of L.P.P.

8) Test the convergence of the sequence whose $n^{th}$ term is $\dfrac{\log(n+1) - \log n}{\tan \frac{1}{n}}$.

P.T.O.

9) Find the limit of the sequence whose $n^{th}$ term is $\sqrt{n^2+1}-n$.

10) Show that the sequence $\{x_n\}$ whose $n^{th}$ term is $\frac{1}{2x+5}$ is monotonically decreasing.

11) Define a bounded sequence.

12) Write the nature of a geometric series.

13) Discuss the nature of the series $\sum_{n=1}^{\infty}(-1)^n n$.

14) Test the convergence of the series $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+...$

15) Let $\sum a_n$ and $\sum b_n$ be two series of positive terms such that

   i) $\sum b_n$ is convergent and

   ii) $a_n \leq kb_n \ \forall \ n$ except perhaps for finite number of terms in the beginning $(k>0)$ then prove that $\sum a_n$ is also convergent.



16) Sum to infinity the series $\frac{1}{7}+\frac{1}{3.7^3}+\frac{1}{5.7^5}+...$

17) Define least upper bound for a function f(x).

18) Write the geometrical interpretation of Cauchy's mean value theorem.

19) Expand secx by Maclaurin's expansion upto the term containing $x^2$.

20) Evaluate $\lim_{x\to 1}\left[(1-x)\tan\frac{\pi x}{2}\right]$.

II. Answer **any two** questions : **(2×5=10)**

1) Prove that a subgroup H of a group G is a normal subgroup of G if and only if the product of two right cosets of H in G is also a right coset of H in G.

2) Let $f: G \rightarrow G'$ be a homomorphism from the group G into a group $G'$ with kernel K then prove that K is a normal subgroup of G.

3) State and prove Cayley's theorem.

4) G is any group and g a fixed element of G. Define $T: G \rightarrow G$ by $T(x) = gxg^{-1}$, show that T is an isomorphism.

III. Answer **any three** questions : **(3×5=15)**

1) Compute all the basic feasible solutions of the system of equation $x + 2y + z = 4$ and $2x + y + 5z = 5$.

2) Solve the following L.P.P.graphically : maximize $z = 3x + 2y$ subjected to the constraints $5x + y \geq 10$, $2x + 2y \geq 12$, $x + 4y \geq 12$, $x, y \geq 0$.

3) Solve the following L.P.P. by simplex method, maximize $Z = x - y + 3z$ subject to the constraints $x + y + z \leq 10$, $2x - z \leq 2$, $2x - 2y + 3z \leq 0$, $x, y, z \geq 0$.

4) Obtain an initial solution for the following transportation problem using Vogel's approximation method.

**Destination**

| | $D_1$ | $D_2$ | $D_3$ | Supply |
|---|---|---|---|---|
| $S_1$ | 2 | 7 | 4 | 5 |
| $S_2$ | 3 | 3 | 1 | 8 |
| $S_3$ | 5 | 4 | 7 | 7 |
| $S_4$ | 1 | 6 | 2 | 14 |
| **Demand** | 7 | 9 | 18 | 34 |



IV. Answer **any two** questions : **(2×5=10)**

1) Discuss the nature of the sequence $\left\{x^{1/n}\right\}$ $x > 0$ where x is any real number.

2) Test the convergence of the sequences whose $n^{th}$ term are

i) $\left(\frac{n-3}{n+2}\right)^{n/3}$

ii) $1 + \cos n\pi$

3) Prove that the sequence whose $n^{th}$ term is $\frac{3n+4}{2n+1}$ is monotonically decreasing and is bounded.

V. Answer **any four** questions : **(4×5=20)**

1) State and prove Raabe's test.

2) Discuss the convergence of the series $\frac{2}{1^2}x + \frac{3^2}{2^3}x^2 + \frac{4^3}{3^4}x^3 + \ldots$

3) Discuss the convergence of the series $\sum \frac{(n!)^2}{(2n)!} x_1^n \; x > 0$.

4) Test the convergence of the series $1 + 2x + 3x^2 + 4x^3 + \ldots$

5) Sum to infinity the series $\frac{11.14}{10.15.20} + \frac{11.14.17}{10.15.20.25} + \ldots$

6) Sum to infinity the series $\sum_{n=1}^{\infty} \frac{n^3 - n + 1}{n!}$



VI. Answer **any three** questions : **(3×5=15)**

1) State and prove Rolle's theorem.

2) Discuss the differentiability of the function f(x) defined by

$$f(x) = \begin{cases} 1 - 3x & \text{for } x < 1 \quad \text{at } x = 1 \\ x - 3 & \text{for } x > 1 \end{cases}$$

3) Verify Rolle's theorem for the function $f(x) = x(x + 3)e^{-x/2}$ in $[-3, 0]$.

4) Evaluate $\lim_{x \to 1} \frac{x^x - x}{1 - x + \log x}$.

5) Obtain Maclaurin's expansion of $\log_e(1 \pm e^x)$ up to the term containing $x^4$.

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